3.4 Completed Notes

3.4: Algorithms for Multiplication and Division

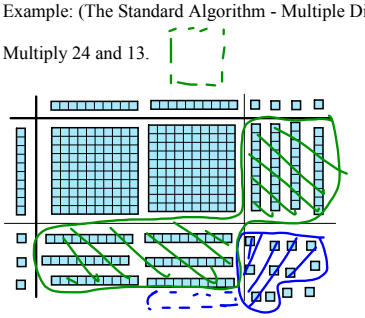
Example: (The Standard Algorithm - Single Digit)

Multiply 3037 and 4.

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 3037 \\ 3037 \\ 3037 \\ +\ 3037 \\ \hline 12148 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ 2} \\ 3037 \\ \times\quad 4 \\ \hline 12148 \end{array}$$

Example: (The Standard Algorithm - Multiple Digit)

Multiply 24 and 13.



Standard Algorithm:

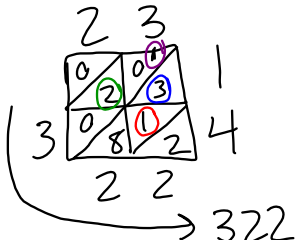
$$\begin{array}{r} {\scriptstyle *} \\ 24 \\ \times\ 13 \\ \hline 72 \\ +240 \\ \hline 312 \end{array}$$

Another Explanation:

$$\begin{array}{r} 23 \\ \times 14 \\ \hline \end{array} = \begin{array}{r} (20+3)(10+4) \\ 20+3 \\ \times 10+4 \\ \hline 80+12 \\ +200+30 \\ \hline 280+42=322 \end{array}$$

Example: (Lattice Multiplication)

(a) Multiply 23 and 14.

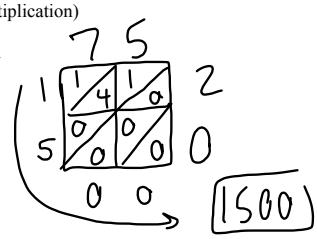


Why does this work?

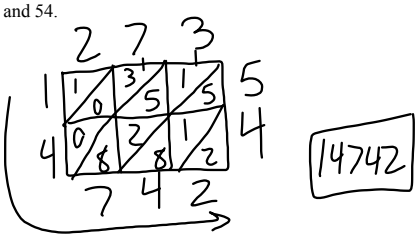
- regrouped 3×4 into 1 ten + 2 units
- actually multiplied 3×10
- actually multiplied 20×10
- regrouped 12 tens into 1 hundred and 2 tens

Example: (Lattice Multiplication)

(b) Multiply 75 and 20.



(c) Multiply 273 and 54.

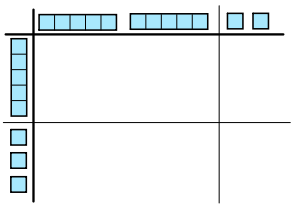


Base 5 Multiplication: We can use both of the previous algorithms with base 5 numbers using this table.

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

3.4 Completed Notes

Example: Multiply 14_{five} and 22_{five}.



Standard Algorithm:

$$\begin{array}{r} {\scriptstyle 1}\\ 14\\ \times\ 22\\ \hline 33\\ +330\\ \hline 413_{five} \end{array}$$

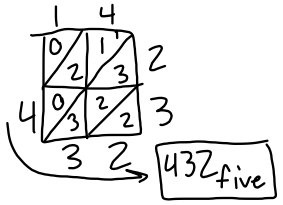
Example: All numbers are base 5.

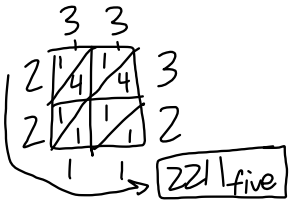
Standard:

$$\begin{array}{r} {\scriptstyle 1}\\ 14\\ \times\ 23\\ \hline 102\\ +330\\ \hline 432_{five} \end{array}$$

$$\begin{array}{r} {\scriptstyle 1}\\ 33\\ \times\ 32\\ \hline 121\\ +2040\\ \hline 2211_{five} \end{array}$$

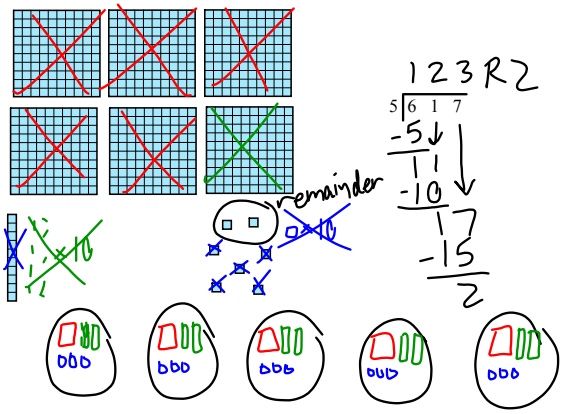
Lattice:





The Long Division Algorithm:

Example: Calculate 617 ÷ 5



$$\begin{array}{r} 123\ R2\\ 5\overline{)617}\\ -5\downarrow\\ \hline 11\\ -10\\ \hline 17\\ -15\\ \hline 2 \end{array}$$

Example: Carefully explain why this works:

$$\begin{array}{r} 86\\ 23\overline{)1986}\\ -184\downarrow\\ \hline 146\\ -138\\ \hline 8 \end{array}$$

① Form 23 groups

② Can't put 1 cube in 23 groups, so form 10 flats, giving 19 flats.

③ Convert 19 flats into 190 longs, giving 198 longs

④ Put 8 longs in each of the groups. You use 184 longs and leave 14 longs.

⑤ Convert the 14 longs into 140 units, giving 146 units.

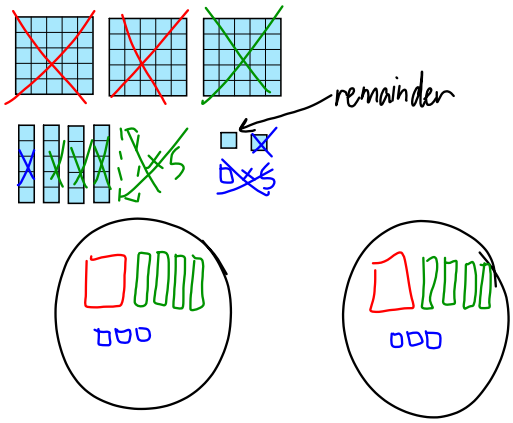
⑥ Put 6 units in each of the 23 groups. You use 138 units and leave the remainder of 8.

Example: Calculate $342_{five} \div 2_{five}$.

$$\begin{array}{r} 143_{five}\ R1_{five}\\ 2\overline{)342}\\ -2\downarrow\\ \hline 14\\ -13\\ \hline 12\\ -11\\ \hline 1 \end{array}$$

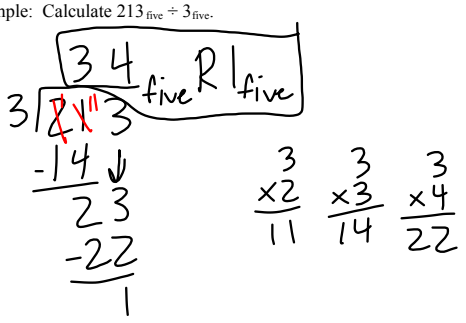
$$\begin{array}{r} 2\\ \times 3\\ \hline 11 \end{array} \qquad \begin{array}{r} 2\\ \times 4\\ \hline 13 \end{array}$$

Let's try $342_{five} \div 2_{five}$ with Base 5 blocks.

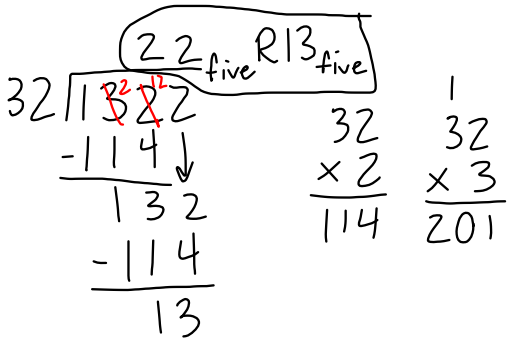


3.4 Completed Notes

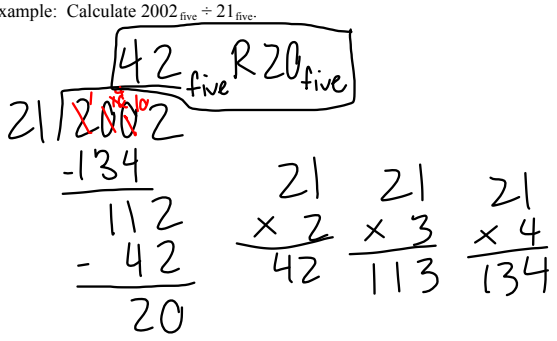
Example: Calculate $213_{five} \div 3_{five}$.



Example: Calculate $1322_{five} \div 32_{five}$.



Example: Calculate $2002_{five} \div 21_{five}$.



$3424_{five} \div 23_{five}$